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Increasing market efficiency in the stock markets

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Abstract. We study the temporal evolutions of three stock markets; Standard and Poor's 500 index, Nikkei 225 Stock Average, and the Korea Composite Stock Price Index. We observe that the probability density function of the log-return has a fat tail but the tail index has been increasing continuously in recent years. We have also found that the variance of the autocorrelation function, the scaling exponent of the standard deviation, and the statistical complexity decrease, but that the entropy density increases as time goes over time. We introduce a modified microscopic spin model and simulate the model to confirm such increasing and decreasing tendencies in statistical quantities. These findings indicate that these three stock markets are becoming more efficient.

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1 Introduction

Econophysics is one of the most active fields in interdisciplinary research [1–19]. Among the many topics of interest to econophysics, the market efficiency of various stock markets has been actively studied [15–21]. The market is said to be efficient if all available information is instantly processed when it reaches the market and is immediately reflected in price adjustments of the assets traded. The efficient market hypothesis is that a market is assumed to be an efficient market [22–24]. Samuelson [25] explicitly formulated the efficient market hypothesis by demonstrating the price process as a stochastic process called a martingale. In an efficient market, price changes are unpredictable from the previous time series. Therefore, the autocorrelation of price changes has to be negligible and their probability distributions are given by Gaussian distribution. However, empirical evidences from a number of real markets do not support the efficient market hypothesis [20,21].

There are many methodologies to analyze financial time series. Observing the probability density functions (PDFs) of the log-return is one of the simplest and most popular methods. Many papers have already been published studying the PDFs of the log-return in stock markets [11–15,26,27]. Especially, the different characteristics between of mature markets and emerging markets [12], the relations between the shape of distributions and time lags [13], market efficiencies [15] and criteria between bubble and anti-bubble [9,11] in the financial markets, have been studied by using the PDFs of the log-return. Another method is computational mechanics [28]. Computational mechanics has been used in various fields of science [29–31], including stock markets [18,19]. Computational mechanics enables us to analyze the complexity and structure in financial markets quantitatively by finding causal structures of the time series [32].

Agent based modeling has also been widely used in the social sciences and econophysics. Agent based models in econophysics have been constructed by using agents clustering [1], Ising-like spin models [2,15], and Potts-like spin models [4]. Simulations on these microscopic agent based models have been performed to explain the shapes of the PDFs depending on traders' characteristics [11] and information flows [15], and speculative activities for bubbles and crashes in stock markets [5]

In this paper, we analyze the time series of Standard and Poor's 500 Index (S&P 500), the Korean Composite Stock Price Index (KOSPI), and the Nikkei 225 Stock Average (NIKKEI) by using the PDFs, the autocorrelation function, the standard deviation, the statistical complexity, and the entropy density of the log-return on these three stock markets. We then introduce and simulate a modified microscopic spin model for the financial markets

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to compare with the results obtained from the empirical data.

2 Empirical data and analysis

We use high frequency (1 min) S&P 500 data for the period of 1983–2006, KOSPI data (1992–2003), and NIKKEI data (1997–2005). S&P 500 and NIKKEI data represent mature markets while the KOSPI data represent an emerging market. We use only intra-day data to exclude the discontinuity jumps between the previous day's closing price and the next day's opening price due to overnight effects. In the computations we use the log-return defined as the change in the logarithm price as follows with the time lag Δt:

$$S_{\Delta t}(t) \equiv \log Y(t + \Delta t) - \log Y(t), \tag{1}$$

where $Y(t)$ is the price of the stock at time t.

2.1 Probability density function

It is broadly assumed that price changes in an efficient market cannot be predicted because the price changes randomly. Assuming the price change to be a random multiplicative process, the probability density functions then take the form of Gaussian distribution. However, the distributions of the log-return are found to be non-Gaussian [13,15,26,27,33]. Empirically, the tails part of the PDFs are wider and the centers of the PDFs are sharper than a Gaussian distribution. It has been reported that the PDFs in mature markets have a power-law tail, while emerging markets have an exponential tail [12,14]. The PDFs also vary with time lag. Price changes have a power-law distribution for a short time lag, and an exponential distribution when the time lag is long. Moreover, for a long time lag, the distribution becomes Gaussian [13,33].

Figure 1a shows the PDFs of S&P 500 log-returns from 1998 to 2005. The shape of the distribution in 1998 in Figure 1a is close to the Lévy distribution and the tail part shows a power-law distribution. However, the shape of the PDFs in the 2000s becomes narrower and the shape of tail part becomes thinner, indicating an obvious change from the Lévy distribution to the exponential distribution. The same behaviors are also observed for KOSPI and NIKKEI data.

This phenomena can be confirmed by the increasing behavior of the tail index, defined as a power-law exponent of the tail part of the PDFs. Figure 1b shows the temporal evolutions of tail index in the PDFs for the S&P 500, KOSPI, and NIKKEI, respectively. The tail index of the three stock markets increases from 1999 to 2003. It is well known that the shape of PDFs depends on the time lag [13,33,34]. Therefore, we can conjecture that the change of the shape of PDFs is related with the change of effective time lag. Because of the improvement of infra for information flow, effective time is becoming shorter over time. To confirm this, in the next subsections we discuss

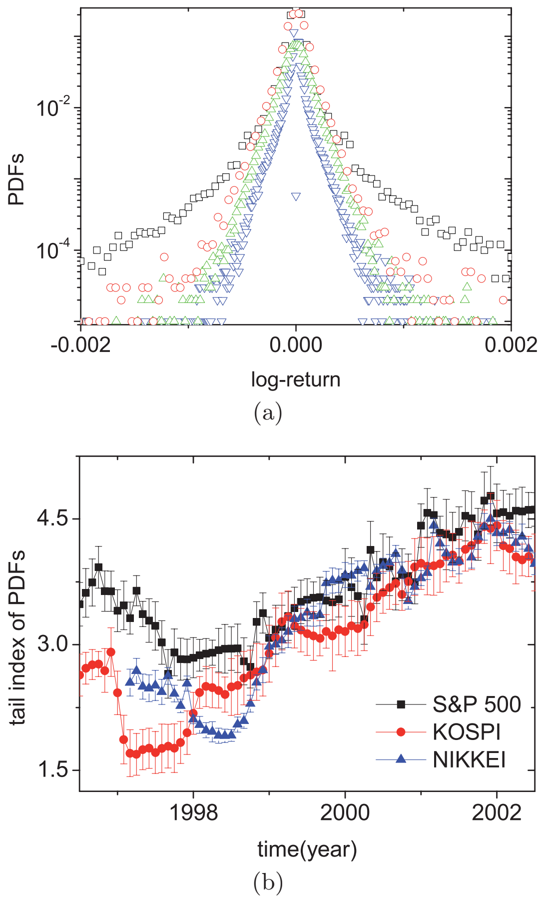


Fig. 1. (a) The PDFs of the log-return for the S&P 500: □ 1998, ○ 1999, △ 2001, and ▽ 2005. (b) Temporal evolutions of the tail index for the S&P 500, KOSPI, and NIKKEI.

the autocorrelation function and the standard deviation to measure long range correlations, and the entropy density and the statistical complexity to measure randomness (or regularity) in the three stock markets.

Tail index is obtained by the least square fitting of the PDFs. The error bars in the figure are asymptotic standard error. We use the χ^2 test to test if a sample of data comes from a population with a power law distribution [10]. For example, the test statistic $\overline{\chi}^2$ is 21.91, and the degree of freedom is 25 for the S&P 500 in 2005. Therefore, the null hypothesis of the power-law tail of the log-return distribution may be accepted with 0.100 of significance level.

2.2 Autocorrelation function and standard deviation

We consider the variance of autocorrelation function to be defined as

$$V_{ACF} = \langle R(\tau)^2 \rangle_\tau, \tag{2}$$

where the autocorrelation function is defined by

$$R(\tau) = \frac{\langle S_1(t) S_1(t+\tau) \rangle_t}{\sigma^2}, \tag{3}$$

and σ is the standard deviation of $S(t)$ (see Eq. (4)). In equations (2) and (3), $\langle \ldots \rangle_{\tau\ (\mathrm{or}\ t)}$ means an average for τ (or t) over the period.

When the time series of the log-return in the stock markets have long range correlations, V_{ACF} has non-zero value, while it approaches zero if there is no correlations. Figure 2a shows the temporal evolution of V_{ACF} which decreases to zero for all three stock markets. V_{ACF} begins to decrease through 1998 and 1999, and becomes almost zero after 1999 for the S&P 500 and NIKKEI, while V_{ACF} decreases over many more years and approaches zero in 2001 for the KOSPI. After 2001, V_{ACF} approaches zero for all three stock markets, meaning that correlations have diminished recently. Therefore, the three stock markets have become more random and market efficiency has increased recently compared with previous years.

We also investigate long range correlations by observing the standard deviation of the log-return [8] defined as

$$\sigma(\Delta t) = \frac{\sqrt{\sum_{i=1}^{n} (\log Y(t_i + \Delta t) - \log Y(t_i))^2}}{\sqrt{n-1}}, \qquad (4)$$

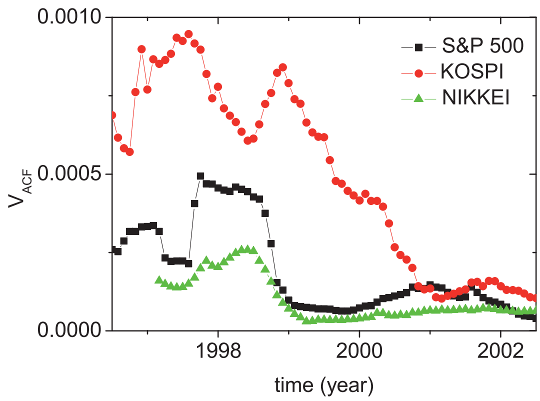
where n is the number of entries in the log-return data and its value is approximately 10^5 for each year. The standard deviation then exhibits the following power-law behavior

$$\sigma(\Delta t) \sim \Delta t^{\mu}. \qquad (5)$$

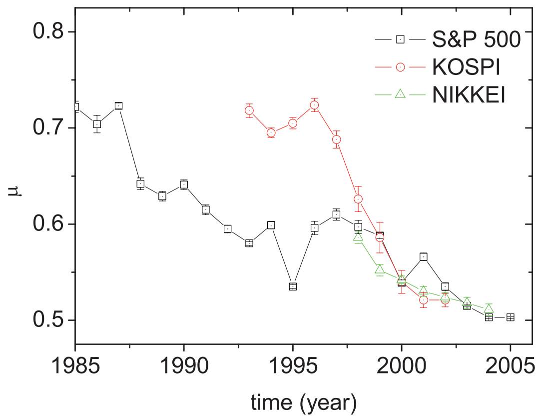
There is a long range correlation if μ is larger than 0.5, no correlation at $\mu = 0.5$, and an anti-correlation for $\mu < 0.5$. The strength of the long range correlations is seen to be larger for less efficient financial markets.

Figure 2b shows the temporal evolutions of μ obtained from the least square fitting of equation (5), and the error bars represent the standard error of empirical data from the fitted equations. The value of μ for S&P 500 data decreases continuously and eventually approaches 0.5, meaning that long range correlations tend to disappear as time goes over time. These decreasing tendencies of the time evolutions of μ show that the stock market becomes more efficient. We also observe the same tendency for the KOSPI and NIKKEI.

We use the χ^2 test to test whether the fitting value of μ is reliable. For example, the p-value for the S&P 500 in 2005 is smaller than 0.0001.



(a)



(b)

Fig. 2. Time evolutions of (a) variance of the autocorrelation functions and (b) scaling exponents of the standard deviation.

2.3 Entropy density and statistical complexity

We also analyze the temporal evolutions of the statistical complexity and the entropy density, by using the causal-state splitting reconstruction algorithm [28] to model the ϵ-machine. We first change the original data $Y(t)$ into the binary time series $F(t)$ as follows:

$$F(t) \equiv \theta(Y(t + \Delta t) - Y(t)), \qquad (6)$$

where $\theta(x)$ is a Heaviside step function and Δt is the time interval which is set to one minute. Then the original data $Y(t)$ are changed into the binary time series $F(t)$ with a countable set $A = \{0, 1\}$, where $F(t)$ is 0 (or 1) when the next index has decreased (or increased).

Next, the probability distribution of a block of L consecutive random variables $X^L = X_i, \ldots, X_{i+L-1}$ is taken as the set of joint probabilities of L consecutive values $\Pr(x^L) = \Pr(x_i, \ldots, x_{i+L-1})$ for all 2^L possibilities. Then the Shannon entropy [35] for the above L block variable X^L is defined as

$$H(L) = -\sum_{x_1 \in A} \cdots \sum_{x_L \in A} \Pr(x_1, ..., x_L) \log_2 \Pr(x_1, ..., x_L), \qquad (7)$$

which measures the uncertainty or randomness in the binary time series $F(t)$. $H(L)$ may diverge as L goes to infinity, because $H(L)$ is a monotonically increasing function of L. Therefore, it is more convenient to introduce the following entropy density

$$h \equiv \lim_{L \to \infty} \frac{H(L)}{L}. \qquad (8)$$

The entropy density for the finite length L can also be written as a function of block length L as follows;

$$h(L) \equiv H(L) - H(L-1). \qquad (9)$$

If L is not large enough to fully detect the structure from the time series, h would overestimate the randomness of time series. Therefore, $h(L)$ converges to h as L increases. In this paper, we set to $L = 7$ which is large enough because $h(L = 7) \approx h(L > 7)$.

To calculate the statistical complexity, the ϵ-machine has to be defined. An infinite string $\overleftrightarrow{X} = \ldots X_{-1}X_0X_1\ldots$ can be divided into two semi-infinite parts such as a future $\overrightarrow{X}$ and a history $\overleftarrow{X}$, where each X_i may take a symbol x_i drawn from a finite countable set A. Then, a causal state is defined as a set of histories that have the same conditional probabilities for all possible future events. ϵ is a function that maps each history to a corresponding causal state $\epsilon(\overleftarrow{x})$,

$$\begin{aligned}\epsilon(\overleftarrow{x}) &= \{\overleftarrow{x}' \mid \Pr(\overrightarrow{X} = \overrightarrow{x} \mid \overleftarrow{X} = \overleftarrow{x}) \\ &= \Pr(\overrightarrow{X} = \overrightarrow{x} \mid \overleftarrow{X} = \overleftarrow{x}'), \forall \overrightarrow{x} \in F\}. \end{aligned} \tag{10}$$

Then, the transition probability $T_{ij}^{(a)}$ denotes the probability of generating a symbol $a \in A$ when making a transition from state s_i to state s_j [36,37]:

$$T_{ij}^{(a)} \equiv \Pr(\overleftarrow{X}a \in s_j \mid \overleftarrow{X} \in s_i). \tag{11}$$

The function ϵ mapping histories to causal states and the labelled transition probabilities $T_{ij}^{(a)}$ constitute an ϵ-machine [36], which represents the computational mechanics underlying a given time series.

From the constructed ϵ-machine, the probability of finding the system in the ith causal state after the ϵ-machine has been running infinitely for each i, $\Pr(s_i)$, can be calculated. The components T_{ij} of the transition matrix $T = \sum_{a\in A} T_{ij}^{(a)}$ give the probability of a transition from state s_i to state s_j. $\Pr(s_i)$ is obtained by solving

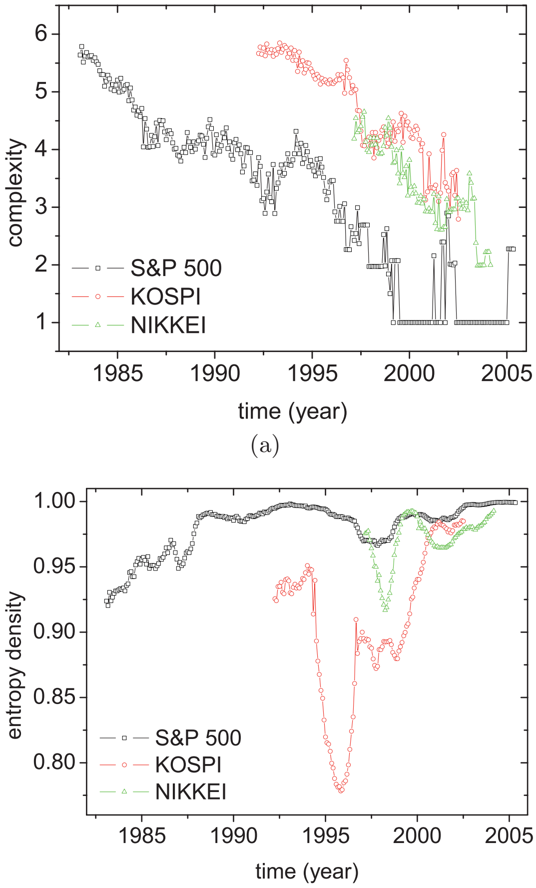
$$\sum_i \Pr(s_i)T_{ij} = \Pr(s_j), \tag{12}$$

and the statistical complexity is defined as

$$C \equiv -\sum_{\{i\}} \Pr(s_i)\log_2 \Pr(s_i). \tag{13}$$

Figure 3 shows the temporal evolutions of the statistical complexity and the entropy density with $L = 7$. The statistical complexity of the S&P 500 data decreases continuously and approaches near 1 or 2 as time goes over time. The time series is considered to become random when the statistical complexity approaches zero.

To support whether the time series is random or not, we also analyze the temporal evolutions of the entropy density. The time series is considered to be regular if the entropy density comes close to zero, and to be random if the entropy density fluctuates around 1. As shown in Figure 3b, the entropy density becomes close to 1 after the 1990s for the S&P 500 and after the 2000s for the KOSPI, while convergency to 1 can not be observed from the NIKKEI due to shortage of data. Though the increasing tendency for the entropy density exists in the S&P 500,



(a) (b)

Fig. 3. Temporal evolutions of (a) the statistical complexity and (b) the entropy density.

it is very weak compared with that of the KOSPI. Hence, we calculate the deviation of the original data from the shuffled data. The deviation between the data and the shuffled data is 7.1×10^{-4}, 7.4×10^{-3}, and 7.5×10^{-4} for the S&P 500, KOSPI, and NIKKEI, respectively. For the S&P 500 and NIKKEI, the deviation is smaller than that of the KOSPI. Moreover, the fluctuation of the entropy density in the mid and late 1990s due to the Asian financial crisis is large, while the statistical complexity is relatively solid compared with the entropy density. From this result, we can conclude that the entropy density is more susceptible to fluctuation in the market and it is hard to verify the trend using the entropy density. The statistical complexity is obtained from the causal states which are calculated by the relation with history of time series stated in equation (10), while the entropy density is the simple probability of configuration of time series. For this reason, the entropy density has the more susceptible property. The original nature of market efficiency is not changed by a shock such as a financial crisis because it is more closely related with the infrastructure for circulation of information in the market. In brief, from temporal evolutions of the statistical complexity and the entropy density, we can conclude that the time series of these three stock markets are growing more random and thus the stock markets are becoming more efficient.

3 The modified model and simulations

As an attempt to account for the observed phenomena, we consider and modify the microscopic spin model [2] of many interacting agents (spins). In the model, agents in the stock market are represented by spins and, interactions between agents and external informations are represented by local fields. This model displays probability distributions of the log-return resembling those of empirical financial time series if the model parameters are chosen properly.

We consider $i = 1, 2, \ldots, N$ agents with orientations $\sigma_i(t) = \pm 1$, corresponding to the decision to buy (+1) or sell (−1) a stock at discrete time-steps t. The orientation of the agent i at time $t + 1$, depends on the local field,

$$I_i(t) = \frac{1}{N}\sum_j A_{ij}(t)\sigma_j(t) + h_i(t), \tag{14}$$

where $A_{ij}(t)$ represents time-dependent interaction strength among agents, and $h_i(t)$ is an external field reflecting the effects of environment. The interaction strength is $A_{ij}(t) = A\xi(t) + a\eta_{ij}(t)$, where $\xi(t)$ and $\eta_{ij}(t)$ are determined randomly in every step. A is an average interaction strength between agents and a is a deviation of the individual interaction strengths from the average. The external field is $h_i(t) = h\zeta_i(t)$, where h is an information diffusion factor and $\zeta_i(t)$ is a random variable influencing the ith agent at time t. The average interaction strength $A\xi(t)$ represents the average reaction of agents to price changes. The terms $a\eta_{ij}(t)$ describes the fluctuating interaction while $h\zeta_i(t)$ describe the fluctuating environment.

From the above local field, the orientation of agents in the next step is determined by

$$\sigma_i(t+1) = \begin{cases} +1 \text{ with probability } p \\ -1 \text{ with probability } 1-p, \end{cases} \tag{15}$$

where $p = 1/(1 + exp\{-2I_i(t)\})$. In this microscopic spin model, the log-return of price changes at time t is given by

$$S(t) = \frac{1}{N}\sum \sigma_i(t). \tag{16}$$

We now want to modify the local field (Eq. (14)) in order to adjust the agent's strategy on stock exchange, depending on the market log-return and the anticipated log-return. We introduce the anticipated log-return of the ith agent $S_i^{ant}(t)$, which is equal to $p \times (+1) + (1 - p) \times (-1)$ with the given local field on the ith agent, leading to

$$S_i^{ant}(t) = \tanh I_i(t). \tag{17}$$

The orientation of the ith agent at time $t+1$ then depends on the following local field with the adjustment

$$I_i^{adj}(t) = I_i(t) + \alpha\left[S(t-1) - S_i^{ant}(t-1)\right], \tag{18}$$

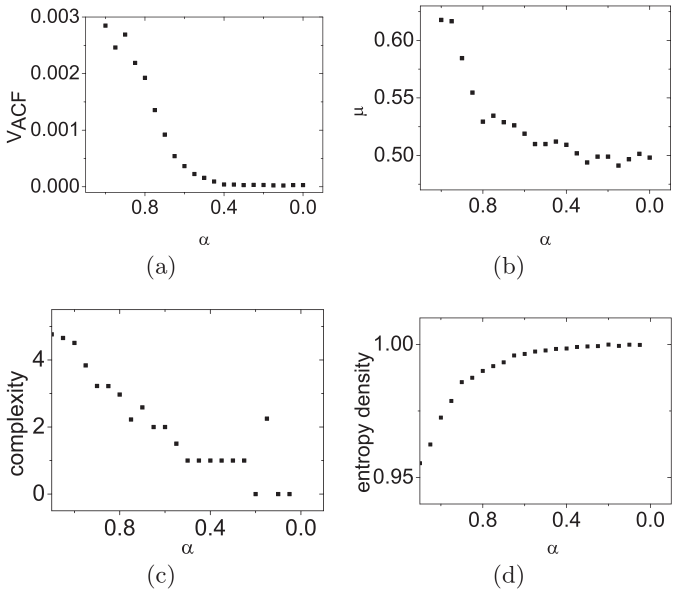


Fig. 4. Dependencies of the value of α; (a) variance of the autocorrelation function, (b) scaling exponent of the standard deviation, (c) the statistical complexity, and (d) the entropy density.

where $\alpha \geq 0$ is the degree of adjustment. For $\alpha = 0$ agents determine their opinions from $I_i(t)$ without any adjustment. When α is non-zero, agents determine their opinions from $I_i^{adj}(t)$ by considering the market log-return and the anticipated log-return. Agents now adjust their opinions by adding or subtracting the difference between the market price changes and the anticipated price changes. The orientation of agents in the next step is now determined by the adjusted local field $I_i^{adj}(t)$ rather than $I_i(t)$.

We simulate this modified model with $N = 1000$ and values of $\xi(t)$, $\eta_{ij}(t)$ and $\zeta_i(t)$ generated within the range [−1, 1]. We then compute the autocorrelation function, the standard deviation, the statistical complexity and the entropy density as a function of the degree of adjustment α.

Figure 4 shows the dependence of four statistical quantities on the value of α. As α decreases from 1 to 0, the variance of the autocorrelation function decreases to 0 (Fig. 4a), the scaling exponent of the standard deviation decreases to 0.5 (Fig. 4b), the statistical complexity decreases to 0 (Fig. 4c), and the entropy density increases to become 1 (Fig. 4d). These decreasing and increasing tendencies are exactly the same as those obtained from the empirical data for the three stock markets. All findings thus support that the stock markets are becoming more efficient.

4 Conclusions

We have studied the time series of stock exchanges using Standard and Poor's 500 Index, the Nikkei 225 Stock Average, and the Korea composite Stock Price Index. We first analyze the temporal evolutions of the probability density function. The PDFs of the log-returns have a fat tail and the tail index is not stationary but increases after the late 1990s.

In order to confirm whether the stock markets become more efficient or not, we have studied the four well-known quantities; the autocorrelation function, the standard deviation, the statistical complexity, and the entropy density for the three markets. As time moves over recent years, the variance of the autocorrelation function decreases and approaches zero, the scaling exponent of the standard deviation continuously decreases and eventually approaches 0.5, the statistical complexity decreases and approaches to near zero, and the entropy density fluctuates around 1. All these results are consistent with increasing market efficiency as the time series become more random, though the entropy density has some problems: not representing the increasing trend clearly for the S&P 500 and NIKKEI, and very susceptible against fluctuation of time series.

We also introduce and simulate the modified microscopic spin model to compare with our findings on the empirical data. All simulations for the same statistical quantities show the identical decreasing and increasing behaviors to support our findings on the empirical data.

In the modified model, the stock market is said to be efficient depending on the degree of adjustment, α. When α is non-zero, the agents adjust their opinions for the stock exchange in the next step. Previously (before 2000), information got around slowly and the market was less efficient so the adjusting behavior was more effective. When α is zero, however, the adjusting behavior is no longer valid so that the agents cannot profit through superiority of information. At present information flow becomes faster and more even because of the rapid development of communication-infra through high speed internet, mobile technologies, and world-wide broadcasting systems. We thus expect contemporary the present stock markets to become more efficient than past markets, confirming the efficient market hypothesis, where a market is efficient if all information is instantly delivered and rapidly reflected by the market prices.

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References

1. V.M. Eguiluz, M. Zimmermann, Phys. Rev. Lett. **85**, 5659 (2000)
2. A. Krawiecki, J.A. Hołyst, D. Helbing, Phys. Rev. Lett. **89**, 158701 (2002)
3. D. Chowdhury, D. Stauffer, Eur. Phys. J. B **8**, 477 (1999)
4. T. Takaishi, Int. J. Mod. Phys. C **16**, 1311 (2005)
5. T. Kaizoji, Physica A **287**, 493 (2000)
6. T. Kaizoji, M. Kaizoji, Adv. Complex Syst. **6**, 303 (2003)
7. O. Kwon, J.-S. Yang, Physica A, `doi:10.1016/j.physa.2008.01.007 (2008)`
8. Z. Palágyi, R.N. Mantegna, Physica A **269**, 132 (1999)
9. T. Kaizoji, S. Bornholdt, Y. Fujiwara, Physica A **316**, 441 (2002)
10. E. Scalas, K. Kim, J. Korean Phys. Soc. **50**, 105 (2007)
11. T. Kaizoji, *Econophysics of Stock and other Markets: Proceedings of the Econophys-Kolkata II Series*, New Economic Windows (Springer, 2006), p. 3
12. K. Matal, M. Pal, H. Salunkay, H.E. Stanley, Europhys. Lett. **66**, 909 (2004)
13. A.C. Silva, R.E. Prange, V.M. Yakovenko, Physica A **344**, 227 (2004)
14. L.C. Miranda, R. Riera, Physica A **297**, 509 (2001)
15. J.-S. Yang, S. Chae, W.-S. Jung, H.-T. Moon, Physica A **363**, 377 (2006)
16. D.O. Cajueiro, B.M. Tabak, Physica A **336**, 521 (2004)
17. K.-P. Lim, Physica A **376**, 445 (2007)
18. J.B. Park, J.W. Lee, J.-S. Yang, H.-H. Jo, Physica A **379**, 179 (2007)
19. J.W. Lee, J.B. Park, H.-H. Jo, J.-S. Yang, H.-T. Moon, e-print `arXiv:physics/0607282`, 2006
20. A.W. Lo, A.C. MacKinlay, Rev. Financ. Stud. **1**, 41 (1988)
21. F. Lillo, J.D. Farmer, e-print `arXiv:cond-mat/0311053`, 2004
22. E.F. Fama, J. Finance **25**, 383 (1970)
23. R.N. Mantegna, H.E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, 2000)
24. L. Tesfatsion, `http://www.econ.iastate.edu/classes/econ 308/tesfatsion/emarketh.htm`
25. P.A. Samuelson, Industrial Management Rev. **6**, 41 (1965)
26. H.E. Stanley, L.A.N. Amaral, X. Gabaix, P. Gopikrishnan, V. Plerou, Physica A **299**, 1 (2001)
27. J.L. McCauley, G.H. Gunaratne, Physica A **329**, 178 (2003)
28. C.R. Shalizi, K.L. Shalizi, J.P. Crutchfield, `arXiv:cs.LG/0210025`, 2002
29. J.E. Hanson, J.P. Crutchfield, Physica D **103**, 169 (1997)
30. C.R. Shalizi, K.L. Shalizi, R. Haslinger, Phys. Rev. Lett. **93**, 11 (2004)
31. J.P. Crutchfield, D.P. Feldman, Phys. Rev. E **55**, 2 (1997)
32. R.W. Clarke, M.P. Freeman, N.W. Watkins, Phys. Rev. E **67**, 016203 (2003)
33. R. Vicente, C.M. de Toledo, V.B.P. Leite, N. Caticha, Physica A **361**, 272 (2006)
34. E. Scalas, Physica A **362**, 225 (2006)
35. C.E. Shannon, Bell Syst. Tech. J. **27**, 379 & 623 (1948)
36. C.R. Shalizi, J.P. Crutchfield, J. Stat. Phys. **104**, 817 (2001)
37. D. Feldman, `http://hornacek.coa.edu/dave/Tutorial/index.html`.